PHYS 1901 Assignment 1

Due: Tuesday, October 17, 2017

DON'T PANIC

Seriously, don't panic. With a bit of effort, everyone in this class can succeed on this assignment. The bark is far worse than the bite. It's a bit challenging, but with purpose (and hopefully it's also a bit fun). This assignment will help you understand the importance of the scientific method and how the application of even a small amount of mathematics can reveal so much about our place in the Universe. Please ask for help if you need it. **Don't delay trying the assignment!**

1. **The Scientific Method and Society:** The problem of climate change represents the intersection of science, economics, and government policy. While scientific discovery forms a necessary input for policy decisions, society must balance risks in an uncertain future. Please read the following article [1] by economist Bob Litterman. 10 marks

   (a) In approximately 500 words, explain Litterman's argument that carbon pricing rests on trade-offs between risks involving economic growth and the risks of catastrophic outcomes.

   (b) What further role do you see for the scientific method in helping to set appropriate carbon abatement policies? What are the limitations of the scientific method in addressing climate change?

2. **The Celestial Sphere:** Suppose that you are standing in a field just outside the city of Ottawa (45°N). For the scenario below, draw and label a basic picture of the celestial sphere as viewed by you at your location. Include the the horizon, the celestial equator, the ecliptic, both celestial poles, the location of the solstices and equinoxes, and the Sun. 10 marks

   (a) It is sunset on the first day of fall.

   (b) **Bonus question (5 marks):** From the position of Inuvik, Northwest Territories, draw the celestial sphere at noon on December 21.

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[1] http://object.cato.org/sites/cato.org/files/serials/files/regulation/2013/6/regulation-v36n2-1-1.pdf

3. **Newton's (apocryphal) apple:** In the popular culture's imagination, Isaac Newton arrived at the Universal Law of Gravitation in a flash of inspiration from being struck on the head by a falling apple while sitting under a tree. Almost certainly the story is apocryphal, but it does contain a (small) kernel of truth. One of Isaac Newton's first biographers, William Stukeley, who was also Newton's friend, recounted the story: 20 marks

   After dinner, the weather being warm, we went into the garden and drank tea under the shade of some apple tree; only he and myself. Amid other discourse, he told me, he was just in the same situation, as when formerly the notion of gravitation came into his mind. Why should that apple always descend perpendicularly to the ground, thought he to himself; occasioned by the fall of an apple, as he sat in contemplative mood.

   Why should it not go sideways, or upwards? But constantly to the Earth's centre? Assuredly the reason is, that the Earth draws it. There must be a drawing power in matter. And the sum of the drawing power in the matter of the Earth must be in the Earth's centre, not in any side of the Earth.

   Therefore does this apple fall perpendicularly or towards the centre? If matter thus draws matter; it must be proportion of its quantity. Therefore the apple draws the Earth, as well as the Earth draws the apple.

   Over time people embellished the details—in which Newton himself played no small part—giving rise to the story we have today. In fact, the idea of an inverse square law for gravity had been floating in the minds of many of Newton's contemporaries, including Robert Hooke, Christopher Wren (the architect of St Paul's Cathedral), and Edmond Halley (see next question), even before Newton wrote the Universal Law of Gravitation. Newton acknowledged that all three men had separately appreciated the inverse square law in his *Philosophiæ Naturalis Principia Mathematica*. Brian Koberlein, an astrophysicist with the Rochester Institute of Technology, has nice brief discussion of the history.

   Newton carefully observed the world and his mathematical genius placed the theory of gravitation firmly within the framework of what we now call Newtonian mechanics. Let's walk in Newton's and his contemporaries' footsteps by taking a journey similar to the one that led to the Universal Law of Gravitation.

   (a) Near the surface of the Earth, objects accelerate at $g = 9.8 \times 10^2$ cm/s^2. Notice the funny units. The units mean that if we were to drop an object, that object would increase its speed by 9.8×10^2 cm/s every second—hence the square in the seconds unit. Recall from class that acceleration refers to the change in velocity, not simply the change in speed. An object in uniform circular motion (the object moving in a circle with constant speed) must be accelerating because the object's direction is continuously changing. Let's calculate the acceleration of an object with uniform circular motion. Look at the diagrams in Figure 1 and Figure 2. We want a relationship between the properties of the circle and the change in the velocity direction over a small period of time, Δt. In Figure 1, we see that at each instant in time, the velocity is tangent to the circle—the object wants to keep going in a straight line, but it can't because some force is causing it to accelerate, keeping it on a circular path. Figure 1 shows the object at two different times with a difference of Δt. The red arrow is a translation of the velocity vector at the later time back to the initial time. We can clearly see that the two velocity vectors are not pointing in the same direction after the small elasped time, Δt. The change in the velocity, Δv, divided by the small change in time, Δt, is the acceleration, $a = \Delta v / \Delta t$. In Figure 1, the change in velocity is the length of the line connecting the tip of the red vector with the tip of the first velocity vector—the length of the base of a triangle. The acceleration of uniform circular motion points to the circle's center which by Newton's Laws, implies that the force must also be center seeking. We need to determine the change in velocity during the elapsed time Δt. In Figure 2, we see that the change in velocity is the length of the red arrow, the base of the triangle that the two velocity vectors make in Figure 1. In the limit of a very short time interval, Δt, the angle, θ, will be very small and the arc length of a circle in Figure 2 with radius v and angle θ will be very close to the length of the base of the triangle. Making the approximation that the arc length of the circle in Figure 2 is the length of the base of the triangle, and recalling from class the relationship between the arc length of a circle and its radius (arc length = radius × angle), the change in the velocity is $\Delta v = v\theta$. But we also know that θ is the angle that opened up as the object went around the circle during the small time interval Δt. The speed going around the circle is $v =$ distance travelled/elapsed time $= r\theta/\Delta t$. Using the relationships in this question and Figures 1 and 2, show that the acceleration of an object going in a circle is:

$$a = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}. \tag{1}$$

The acceleration of uniform cicular motion is called centripetal acceleration (centripetal means center seeking).

(b) Assume that the moon moves in a circular orbit around the Earth with a radius of $R_{\text{moon-orbit}} = 3.8 \times 10^{10}$ cm with a period of 27 days. What is the speed of

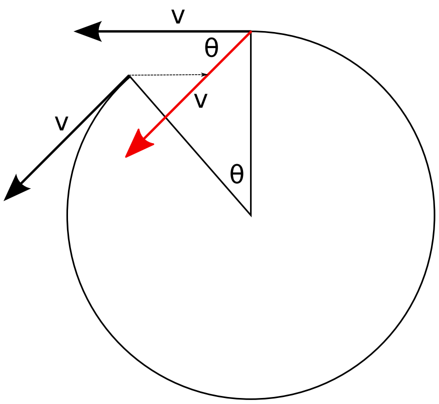

Figure 1: Centripetal acceleration. The red velocity vector is the translation back to the velocity vector after the elapsed time back to the initial time. The dashed arrow represents the effect of the translation.

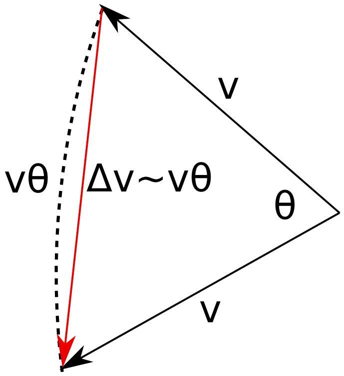

Figure 2: The change in the velocity is the length of the base of the triangle, the red vector. We can approximate the length of the base by the arc length of the circle with radius v: base $\approx v\theta$.

the moon around the Earth in cm/s? (Hint: What is the circumference of the moon's orbit in centimetres? How many seconds are there in 27 days?)

(c) What is the centripetal acceleration, $a_{\text{moon}} = v^2_{\text{moon}}/R_{\text{moon-orbit}}$, of the moon in cm/s^2?

(d) Calculate the ratio of the acceleration of falling objects at the Earth's surface, g, to the moon's centripetal acceleration: g/a_{moon}. Now, calculate the square of the ratio of the Earth-moon distance to the square of the Earth's radius: $R^2_{\text{moon-orbit}}/R^2_{\text{Earth}}$. The radius of the Earth is $R_{\text{Earth}} = 6.4 \times 10^8$ cm. Compare the two ratios. Do you see how Newton and his contemporaries realized that the Universal Law of Gravitation had to be an inverse square force?

(e) We can now see the origin of Kepler's Third Law, at least for circular orbits. The speed around a circular orbit is $v = 2\pi r/P$ where P is the period. This equation is just the total distance, the circumference of the circle, divided by the time it takes to go around, the period. But we just reasoned that gravity is a center seeking inverse square force, so the centripetal acceleration has to equal the acceleration from gravity,

$$\frac{v^2}{r} = \frac{K}{r^2}, \tag{2}$$

for some constant of proportionality, K. (The constant of proportionality is GM where G is Newton's constant and M is the mass of the parent body.) Using $v = 2\pi r/P$, show that

$$P^2 = \frac{4\pi^2 r^3}{K}, \tag{3}$$

which is Kelper's Third Law: $P^2 \propto r^3$!

4. **Halley's comet and dropping a penny from the Earth to the Sun:** Edmond Halley, known today for his eponymous comet, was a contemporary of Newton's. A keen astronomer, Halley had a deep interest in gravitational problems and celestial mechanics. In 1684, Edmond Halley travelled to Cambridge to discuss with Newton the problem of establishing a mathematical proof of Kepler's Laws. Upon arriving, Newton told him that he had solved the problem years earlier, but had not bothered to publish the results. Halley asked to see Netwon's notes but Newton had lost them. Newton promised a new solution, which he later forwarded to Halley in a short treatise called, *On the motion of bodies in an orbit*. Halley immediately recognized the importance of Newton's work and returned to Cambridge to help Newton publish his findings. Newton expanded the manuscript into a new treatise, *Philosophiæ Naturalis Principia Mathematica*, which Halley published at his own expense in 1687. Principia went on to become one of the most influential books in history.

15 marks

Halley noticed that a comet which appeared in 1682 had similar properties to two other comets observed in 1531 and 1607 respectively. Halley conjectured that all three observations were in fact the same celestial body and he calculated the comet's orbital properties using Newton's ideas. Although he did not live to see it, Halley's conjecture proved correct—his comet returned in 1758, reflecting its 76 year orbital period.

Let us use Kepler's Laws to learn more about Halley's comet.

(a) Based on Kelper's Third Law, $P^2 \propto a^3$, and Halley's comet's 76 year orbital period, calculate the length of the comet's semi-major axis in Astronomical Units.

(b) Halley's comet's closest approach to the Sun (called the perihelion) is 0.5 AU. Given the length of the semi-major axis you computed in (a), what is Halley's comet's furthest distance from the Sun (called the aphelion)? Refer to diagram **(a)** in Figure 3. Notice the extreme elliptical nature (high eccentricity) of Halley's comet's orbit.

(c) When visiting the CN Tower in Toronto, people often wonder what would happen if you dropped a penny from the top (answer: not much!). Let's go big—instead of dropping a penny from the CN Tower, let's imagine dropping a penny from the Earth and watch it fall to the Sun. How long will it take for the penny to reach the Sun? Our work with Halley's comet can help us answer this question. In your analysis of Halley's comet, notice that the aphelion distance is almost twice the length of the semi-major axis. Imagine dropping the

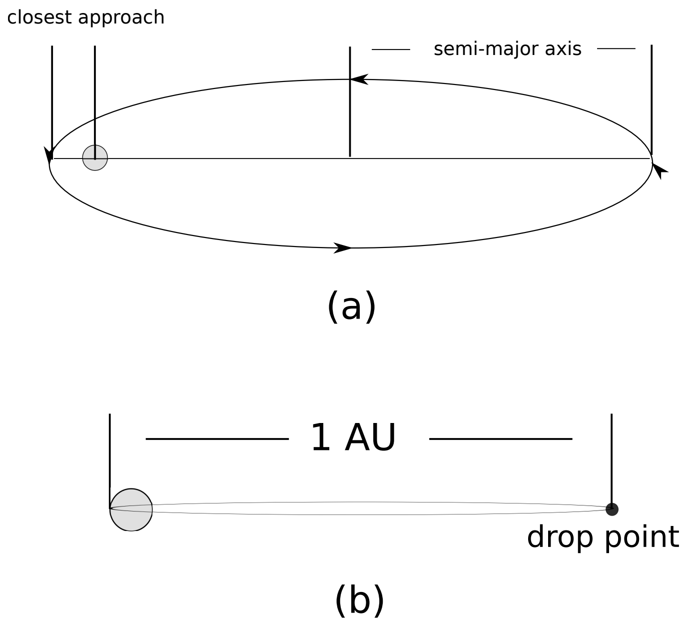

Figure 3: (a) A repesentation of Halley's comet's orbit. Notice the definition of the semi-major axis—it is half the long axis (major axis) of the ellipse. (b) Dropping a penny from a platform at the Earth-Sun distance with a tiny sideways velocity. Notice that the elliptical orbit almost becomes a straight line.

penny from the Earth so that it has just a little sideways velocity. The resulting orbit would be an ellipse with very high eccentricity. Refer to Figure 3 **(b)** for a picture of the idea. Half the period of such an orbit would be the time it would take for the penny to reach the Sun. Use this orbit to calculate the drop time. (Hint: What is the semi-major axis of the orbit in Figure 3 **(b)**?)

55 marks

I THINK YOU SHOULD BE MORE SPECIFIC HERE IN STEP TWO
THEN A MIRACLE OCCURS . . .